NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2978

AN APPROXIMATE METHOD OF CALCULATING THE DEFORMATIONS OF WINGS HAVING SWEPT, M OR W, Λ, AND SWEPT-TIP PLAN FORMS

By George W. Zender and William A. Brooks, Jr.

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

An approximate method of calculating the deformations of wings of uniform thickness having swept, M or W, Λ, and swept-tip plan forms is presented. The method employs an adjustment to the elementary beam theory to account for the effect of the triangular root portion of a swept wing on the deformation of the outboard section of the wing. To demonstrate the general applicability of the method, the modified elementary theory is applied to the more complex M or W, Λ, and swept-tip plan forms as well as to swept plan forms. For the purpose of calculating angles of attack, it is shown that the unmodified elementary beam theory applied to that part of the wing outboard of the root triangle produces satisfactory results. However, for calculating deflections it is necessary to include the effects of the root-triangle deformation.

INTRODUCTION

For many aeroelastic problems it is desirable to have a fairly simple relation between the loads and deformations (particularly the angles of attack) of the wing. However, the problem of analyzing the deformations of swept wings is inherently complex unless simplifying assumptions are made. For the most part, therefore, in aeroelastic problems the swept wing is idealized into a straight beam and the elementary theory of beams applied. The straight beam is assumed to be clamped at an "effective root" location which is determined empirically in order to achieve good comparisons with experimentally determined distortions. In most cases the effective root turns out to be located somewhere within the triangular root section (fig. 1).

One important disadvantage of the effective-root concept is that the root location varies - for example, with a given loading, the best root location for calculating deflections is not the same as that for calculating angles of attack; in addition a similar variation with the type of loading also occurs. Another approximate procedure, which does not have these difficulties, is suggested by the experimental observation that, for reasonably large length-width ratios (on the order of 2 or more), the portion of the wing outboard of the triangular root section (fig. 1) behaves like an ordinary cantilever beam and that the coupling of bending and torsion inherent in a swept wing seems to take place primarily within the root triangle. In this procedure, the distortions of the outboard portion are computed by elementary beam theory and are added to the distortions contributed by the triangular root section.

The success of this procedure hinges on whether a simple method for analyzing the triangular section that yields good comparisons with experiment can be found. The purpose of the present paper is to show that such a method does exist for solid wings of uniform thickness and to demonstrate its accuracy by application not only to swept wings but also to the more complex M or W, Λ, and swept-tip plan forms (fig. 2). Although the development herein pertains to solid wings of uniform thickness, the procedure of treating the outboard portion of the wing as an ordinary cantilever beam on which the distortions of the root triangle are superposed should also be valid for solid wings with nonuniform thickness and for built-up wings.

The procedure of treating the outboard portion of the wing as an ordinary cantilever beam on which the distortions of the root triangle are superposed is also presented in reference 1. The essential difference between the present paper and reference 1 is in the method of calculating the distortions of the triangular root portion of the wing.

SYMBOLS

a_n	coefficients in series for w
α	angle of attack, rotation in plane parallel to clamped root
$\overline{\alpha}$	average rotation α at $x = c \sin \Lambda$
β	rotation in plane perpendicular to clamped root
$\overline{\beta}$	average rotation β at $x = c \sin \Lambda$
c	root chord (see appendix)
Λ	sweep angle, deg

D	flexural rigidity, $\frac{Et^3}{12(1 - \mu^2)}$
E	Young's modulus
I	moment of inertia of cross section parallel to structural chord, $\frac{c \cos \Lambda t^3}{12}$
l	distance to load (see fig. 3)
L	length of outer panel (see fig. 1)
M	bending moment in plane parallel to x-axis, in-lb
m, n, j	integers
μ	Poisson's ratio
P	shear load
t	thickness
T	twisting moment in plane parallel to y-axis
T_ξ	twisting moment in plane parallel to η-axis
U	potential energy
w	deflection in z-direction (see fig. 1)
$\bar{w}$	average deflection w at $x = c \sin \Lambda$
x, y, z; ξ, η	coordinates of point measured from origin at intersection of root and leading edge of wing (see appendix)

Subscript:

elem	elementary

METHOD OF ANALYSIS

As was mentioned in the "Introduction", the solution to the problem of finding the deformations of solid wings of uniform thickness having a swept plan form is, in the present paper, based on the assumption that interaction between the root triangle and the outboard portion of the wing can be neglected. Elementary beam theory is applied to the outboard section of the wing, and the resulting deformations plus those contributed by the triangular root section constitute the total deflections. An approximate solution (based on the experimental observation of negligible angle-of-attack change in the root triangle due to wing loading) for the deformation of the root triangle is given in the appendix.

Swept Wings

In the treatment of a swept wing, the wing is considered by parts as indicated in figure 1. By use of the equations shown in figure 3, elementary beam theory is applied to the outboard portion of the wing. Simple equations for the deflection $\overline{w}$ and the rotation $\overline{\beta}$ of the root triangle (fig. 1) are given in the appendix for various loadings. The distortions $\overline{w}$ and $\overline{\beta}$ have a marked effect on the outboard deflections for most types of loading. However, the angle of attack α is not affected by $\overline{w}$ and $\overline{\beta}$, and, therefore, only the elementary beam theory is necessary to evaluate angles of attack of wings having swept, M or W, Λ, and swept-tip plan forms. The results of the foregoing observation are very convenient for those aeroelastic problems where the effects of the wing deflections are relatively small as compared with the effects of the angles of attack. For streamwise torque, both the deflection $\overline{w}$ and the rotation $\overline{\beta}$ of the root triangle are zero, as shown in the appendix, and, therefore, not only the angle of attack α of the outboard part but also the deflection w is obtained from the elementary theory alone.

Wings Having M or W Plan Form

For the purpose of calculating the deflections and angles of attack, the M or W wing is separated as shown in figure 4. It is to be noted that the inner quarter-span is treated in the same manner as previously shown (fig. 1) for the swept models. The outer quarter-span of the M or W model is analyzed by the two methods indicated in figure 4. For the method identified as theory 1, the outer portion is idealized as an unswept cantilever beam of length equal to the center-line length of the outer portion. For the method identified as theory 2, the outer quarter-span is treated in the same manner as the inner quarter-span: that is, identical to the manner in which the swept wing is treated. In either method, after the deformations of each of the separate parts are found, they are all summed in the proper manner to produce the total deformations.

Wings Having Λ and Swept-Tip Plan Forms

The deformations of the Λ- plan-form wing are obtained in the same manner as shown for theory 1 (fig. 4) of the M- or W-plan-form wing. The inner portion of the Λ wing is treated as previously shown for the swept wing, whereas the outer portion is treated as an unswept cantilever beam. The swept-tip wing has a straight beam for the inner portion, whereas the outer portion may be analyzed by either theory 1 or 2 in the same manner that the outboard portion of the M or W wing is treated. Proper summation of the individual deflections and rotations again produces the total deflection and rotation at each spanwise station of the wing.

TEST SPECIMENS AND METHOD OF TESTING

The models tested in this investigation were of the type shown in figure 2; a photograph of a typical test setup is shown in figure 5. The models had uniform thickness and a 30°, 45°, or 60° sweep angle, were made of 24S-T4 aluminum alloy, and were full-span models with a center section. The center section of each model was clamped between two flat bearing blocks, and bending or twisting loads symmetrical to the longitudinal center line were applied to the model.

The deflections were measured with dial indicators of 0.0001-inch least division. In order to eliminate the dial-indicator spring forces, the gages were mounted in the reverse position. A threaded sleeve (see fig. 5) containing a thumbscrew was placed on the indicator in such a manner that the screw was bearing against the end of the spindle. By means of the screw, the spindle was moved against the spring pressure until the reverse end of the spindle engaged the specimen. A "magic eye" electronic device (ref. 2) was used in conjunction with the gages to indicate contact of the gage spindle with the model. Generally, deflection data were taken at points on one span only, but in many cases data at several corresponding points on the opposite span were obtained as a check.

RESULTS AND DISCUSSION

Swept Models

The deflections w and angles of attack α of the swept models are shown in figure 6 for concentrated lift loads applied at the tip and midway between the root and the tip. The curves represent theoretical results, and the symbols show the test data. In figure 6 (as well as figs. 7 and 8), the data for the 60° model appear only for loads applied midway between the root and tip. Actually the load was applied at the tip of the 60° model; however, since the span of this model was one-half that of the 30° and 45° models, it was more desirable for comparison purposes to locate the 60° tip-load data with the 30° and 45° half-span data. The plots of the experimentally obtained angles of attack indicate that the angles of attack due to loading are negligible within the root triangle. It can be seen that there is good agreement between experiment and theory for both deflections and angles of attack.

Figure 7 presents a comparison between theory and experiment for the swept models loaded with pure torque. These data show better agreement for the angles of attack than for the deflections. Again the angle-of-attack test data pointedly indicate negligible angle of attack in the region of the root triangle.

Figure 8 compares the theory and test for swept models loaded with streamwise torque. For this loading, which produces a coupled bending moment and twisting moment, there is exceptionally good agreement between the test results and theory. The theory indicates that for streamwise torque the root triangle may be considered as rigid; therefore, the deflections and twists are obtained from the elementary beam theory.

The significant assumption in the development of the equations of appendix A for solid wings is that the angle of attack α of the root triangle is negligible and, as a consequence, the angles of attack of the outboard portion are obtained simply from elementary theory. It has been demonstrated that this assumption is valid in the case of the solid models and from the experimental data presented in references 3 and 4 it is possible to obtain information regarding the applicability of this assumption for a box-type 45° swept wing. The experimental angles of attack of the box beam are compared in figure 9 with the results obtained from elementary theory for the symmetrical and antisymmetrical bending and twisting loads. For the bending loads, the shear deformation of the spar webs is included in the same manner as given by equations (A2) of references 3 and 4. It is observed that the application of elementary theory and the assumption of 0° angle of attack for the root triangle gives good agreement for the angles of attack of the outboard portion of the box beam for the two torsion cases but is conservative for the two bending cases. It is also observed in the bending cases that experiment and theory differ by a constant amount which may be attributed to the contribution of the root triangle. This angle-of-attack contribution of the root triangle is largely due to cross-sectional warping which, although negligible in the case of solid cross sections, may be appreciable when box-type wings are considered.

Models Having M or W Plan Forms

The deformations of the M or W models loaded with tip lift load and tip streamwise torque are given in figures 10 and 11, respectively. In general, comparison of the experimental values with results obtained by theories 1 and 2 indicates that better agreement was obtained with theory 2. However, it is also apparent that the difference between the two theories, as well as the difference between theory and experiment, decreases as the sweep angle becomes smaller. This result is to be expected since, for zero sweep of the outer portion of the model, the two theories become identical. In effect, theory 2 introduces more stiffness at the junction of the outer and inner parts than does theory 1.

The short horizontal part of the angle-of-attack curves (figs. 10 and 11) of theory 2 results from assuming that the triangle at the junction of the outer and inner wing portions (fig. 4) may be treated in the same manner as the root triangle: namely, that the angle-of-attack changes due to loading are negligible.

Models Having Λ and Swept-Tip Plan Forms

The deflections and angles of attack of the Λ and swept-tip models loaded with a tip lift load and tip streamwise torque are given in figures 12 and 13, respectively. Inasmuch as theories 1 and 2 are identical when the outer portion of the wing is not swept, there is only one theoretical curve given for the Λ model on each plot. The test data for the swept-tip model are bracketed by the two theories; this fact indicates that the stiffness at the juncture of the inner and outer portions is somewhere between the stiffness assumed by the two theories.

CONCLUDING REMARKS

A method is presented by which the deformations of solid wings of uniform thickness having swept, M or W, Λ, and swept-tip plan forms may be approximated. For the purpose of calculating angles of attack, the elementary beam theory applied to the portion of the wing outboard of the triangular root region suffices as the angles of attack of the root triangle are negligible for the loads considered. However, the deflection of the root triangle must be taken into account in order to analyze the deflections of the wings properly. Although, for all cases considered, the agreement between experiment and theory is satisfactory, the agreement is, in general, better for swept, Λ, and swept-tip wings than for M and W wings. The theory as applied to M or W wings becomes more accurate as the sweep angle becomes smaller. In spite of the fact that in the present paper only wings of uniform thickness are discussed, it seems reasonable that with appropriate modifications the method might apply to wings which do not have constant thickness and to wings with built-up construction.

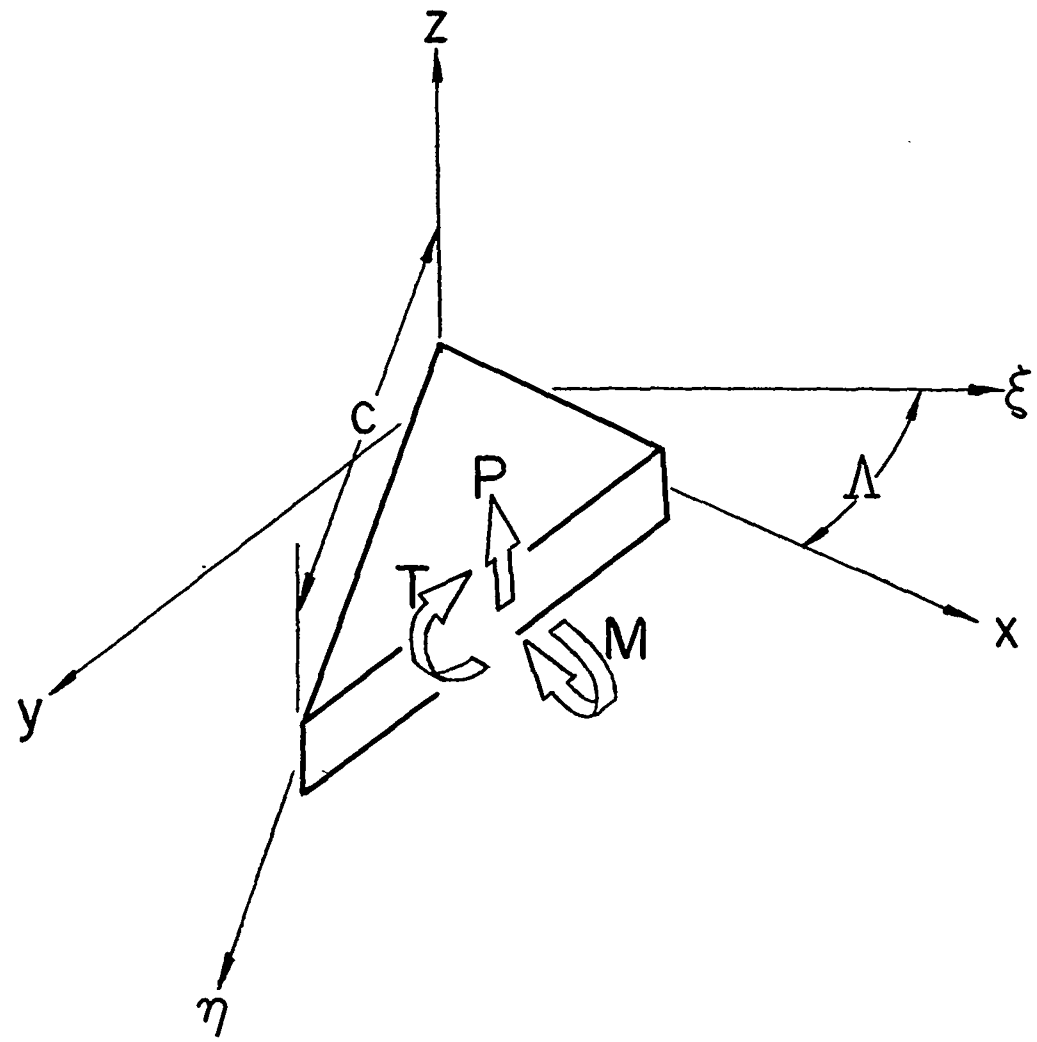
Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 21, 1953.

APPENDIX

ROOT-TRIANGLE DEFORMATION

General Theoretical Analysis

In keeping with the use of an elementary approach to find the deformation of the outboard portion of the wing, an approximate method of finding the deformation of the root triangle is presented. The structure analyzed, the coordinate systems, the loads considered, and the positive direction of the loads are given in the following sketch:



Experimental data revealed that the magnitude of the rotation α of the root triangle was negligible and could be assumed to be zero. It is, therefore, assumed that the deflections are independent of η; w is a function only of ξ. The problem could be solved by using beam theory (wherein, as a result of assumed cylindrical curvature, E is replaced by $E/(1 - \mu^2)$) and integrating the equation for the deflection curve. However, in order to make the method independent of the choice of the assumed deflection function, the minimum-potential-energy principle is employed.

In order to provide greater facility of integrating the energy expression, the relation

$$\xi = x \cos \Lambda - y \sin \Lambda \tag{A1}$$

is used to transform a power series in ξ into the series

$$w = \sum_{n=2}^{\infty} a_n \left(y - \frac{x}{\tan \Lambda}\right)^n \tag{A2}$$

which satisfies the root boundary conditions on the deflections and permits no angle of attack α.

When the shear, moment, and torque resulting from loads on the outboard portion of the wing are evaluated at point 0 $\left(x = c \sin \Lambda,\ y = \frac{c}{2} \cos \Lambda\right)$ and distributed along the outboard edge of the root triangle, the following potential-energy expression is obtained:

$$U = \int_0^{c \sin \Lambda} \int_0^{x/\tan\Lambda} \frac{D}{2} \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1 - \mu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \, \partial y} \right)^2 \right] \right\} dx\, dy + \int_0^{c \cos \Lambda} \frac{M}{c \cos \Lambda} \left(\frac{\partial w}{\partial x} \right)_{x=c \sin \Lambda} dy - \int_0^{c \cos \Lambda} \frac{T}{c \cos \Lambda} \left(\frac{\partial w}{\partial y} \right)_{x=c \sin \Lambda} dy - \int_0^{c \cos \Lambda} \frac{P}{c \cos \Lambda} (w)_{x=c \sin \Lambda} dy \tag{A3}$$

Substitution of the deflection function into the potential-energy equation (A3) and subsequent integration yields the relation

$$U = \frac{D}{2 \sin^4\Lambda} \sum_{m=2}^{\infty} \sum_{n=2}^{\infty} B_{mn} a_m a_n + \left(\frac{M}{\tan \Lambda} + T\right) \sum_{n=2}^{\infty} (-1)^n (c \cos \Lambda)^{n-1} a_n +$$

$$P \sum_{n=2}^{\infty} \frac{(-1)^{n+1} (c \cos \Lambda)^n}{n + 1} a_n \tag{A4}$$

where

$$B_{mn} = \frac{(-1)^{m+n-2} mn(m - 1)(n - 1)}{(m + n - 3)(m + n - 2)} c^{m+n-2} \sin \Lambda(\cos \Lambda)^{m+n-3} \tag{A5}$$

Minimizing the potential energy (equation (A4)) with respect to the unknown coefficients gives

$$\frac{D}{\sin^4\Lambda} \sum_{m=2}^{\infty} B_{mj} a_m + (-1)^j \left(\frac{M}{\tan \Lambda} + T\right)(c \cos \Lambda)^{j-1} -$$

$$(-1)^j \frac{P}{j + 1}(c \cos \Lambda)^j = 0 \tag{A6}$$

from which it is found that

$$\left.\begin{aligned}
a_2 &= \frac{\left[Pc \cos \Lambda - 2\left(\frac{M}{\tan \Lambda} + T\right)\right] \sin^3\Lambda}{4Dc} \\
a_3 &= \frac{P \sin^3\Lambda}{12Dc} \\
a_4, a_5, a_6, &\ldots = 0
\end{aligned}\right\} \tag{A7}$$

The closed form of solution is to be expected in view of the fact that beam theory could have been used rather than the minimum-potential-energy principle.

Thus, the deflection of the root triangle is

$$w = a_2\left(y - \frac{x}{\tan \Lambda}\right)^2 + a_3\left(y - \frac{x}{\tan \Lambda}\right)^3 \tag{A8}$$

The average deflection at $x = c \sin \Lambda$ is

$$\bar{w} = \frac{\int_0^{c \cos \Lambda} (w)_{x=c \sin \Lambda}\, dy}{c \cos \Lambda} \tag{A9}$$

Substitution of equation (A8) into equation (A9) yields

$$\bar{w} = (c \cos \Lambda)^2\left(\frac{a_2}{3} - c \cos \Lambda \frac{a_3}{4}\right) \tag{A10}$$

The required rotation is

$$\bar{\beta} = \frac{\int_0^{c \cos \Lambda} \left(\frac{\partial w}{\partial \xi}\right)_{x=c \sin \Lambda} dy}{c \cos \Lambda} \tag{A11}$$

In general,

$$\frac{\partial w}{\partial \xi} = \frac{\partial w}{\partial x} \cos \Lambda - \frac{\partial w}{\partial y} \sin \Lambda \tag{A12}$$

Substitution of the deflection function into equation (A12) gives

$$\frac{\partial w}{\partial \xi} = \frac{1}{\cos \Lambda} \frac{\partial w}{\partial x} \tag{A13}$$

By using equations (A8) and (A13) with equation (A11),

$$\bar{\beta} = \frac{c}{\tan \Lambda}\left(a_2 - c \cos \Lambda\, a_3\right) \tag{A14}$$

Pure Bending or Pure Torque

In the absence of shear, from an inspection of equation (A7), it is evident that

$$\left.\begin{aligned} a_2 &= -\frac{\left(\frac{M}{\tan \Lambda} + T\right)\sin^3\Lambda}{2Dc} \\ a_3 &= 0 \end{aligned}\right\} \tag{A15}$$

Thus, the desired deflection (eq. (A10)) and rotation (eq. (A14)) become

$$\overline{w} = -\frac{\left(\frac{M}{\tan \Lambda} + T\right)c \sin^3\Lambda \cos^2\Lambda}{6D} \tag{A16}$$

$$\overline{\beta} = -\frac{\left(\frac{M}{\tan \Lambda} + T\right)\sin^2\Lambda \cos \Lambda}{2D} \tag{A17}$$

When there is either pure bending or pure torque, equations (A16) and (A17) are further simplified.

Streamwise Torque

A streamwise torque T_ξ applied to the wing produces combined moment and torque whose magnitudes are

$$\left.\begin{aligned} T &= T_\xi \cos \Lambda \\ M &= -T_\xi \sin \Lambda \end{aligned}\right\} \tag{A18}$$

If there is no shear, substitution of equation (A18) into equation (A7) reveals that

$$a_2 = 0$$

$$a_3 = 0$$

Therefore, the root triangle may be considered as a rigid body, or

$$\left.\begin{aligned}\bar{w} &= 0\\ \bar{\beta} &= 0\end{aligned}\right\} \tag{A19}$$

Shear Load and Moment

If a concentrated force is located on the elastic axis of the outboard portion of the wing, the root triangle will be subjected to shear and bending moment only. Upon evaluation of the coefficients a_2 and a_3, equations (A10) and (A14) become

$$\bar{w} = \frac{c \sin^3\Lambda \cos^2\Lambda}{6D}\left(\frac{3}{8} Pc \cos \Lambda - \frac{M}{\tan \Lambda}\right) \tag{A20}$$

and

$$\bar{\beta} = \frac{\sin^2\Lambda \cos \Lambda}{2D}\left(\frac{Pc \cos \Lambda}{3} - \frac{M}{\tan \Lambda}\right) \tag{A21}$$

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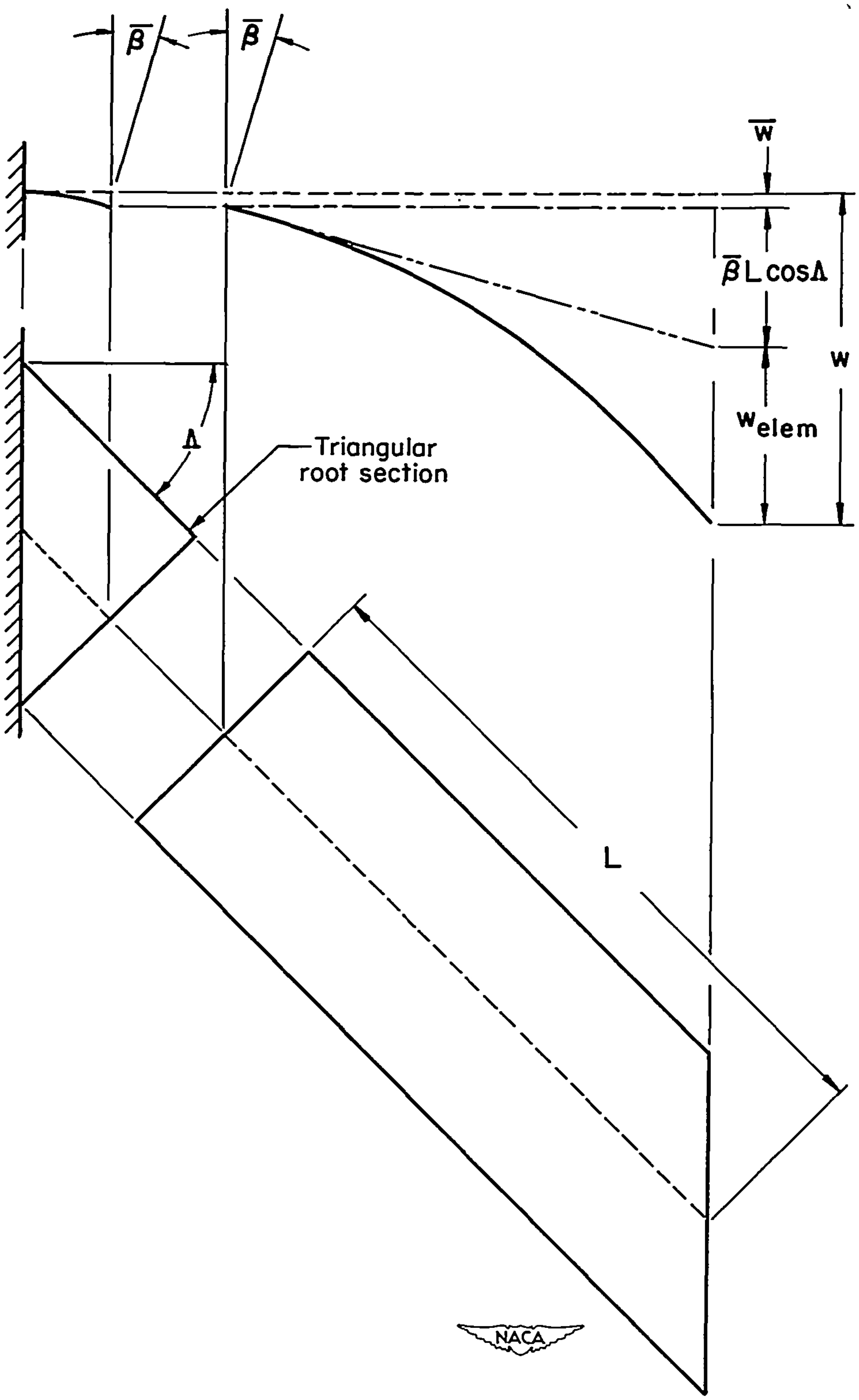


Figure 1.- Separation of swept model.

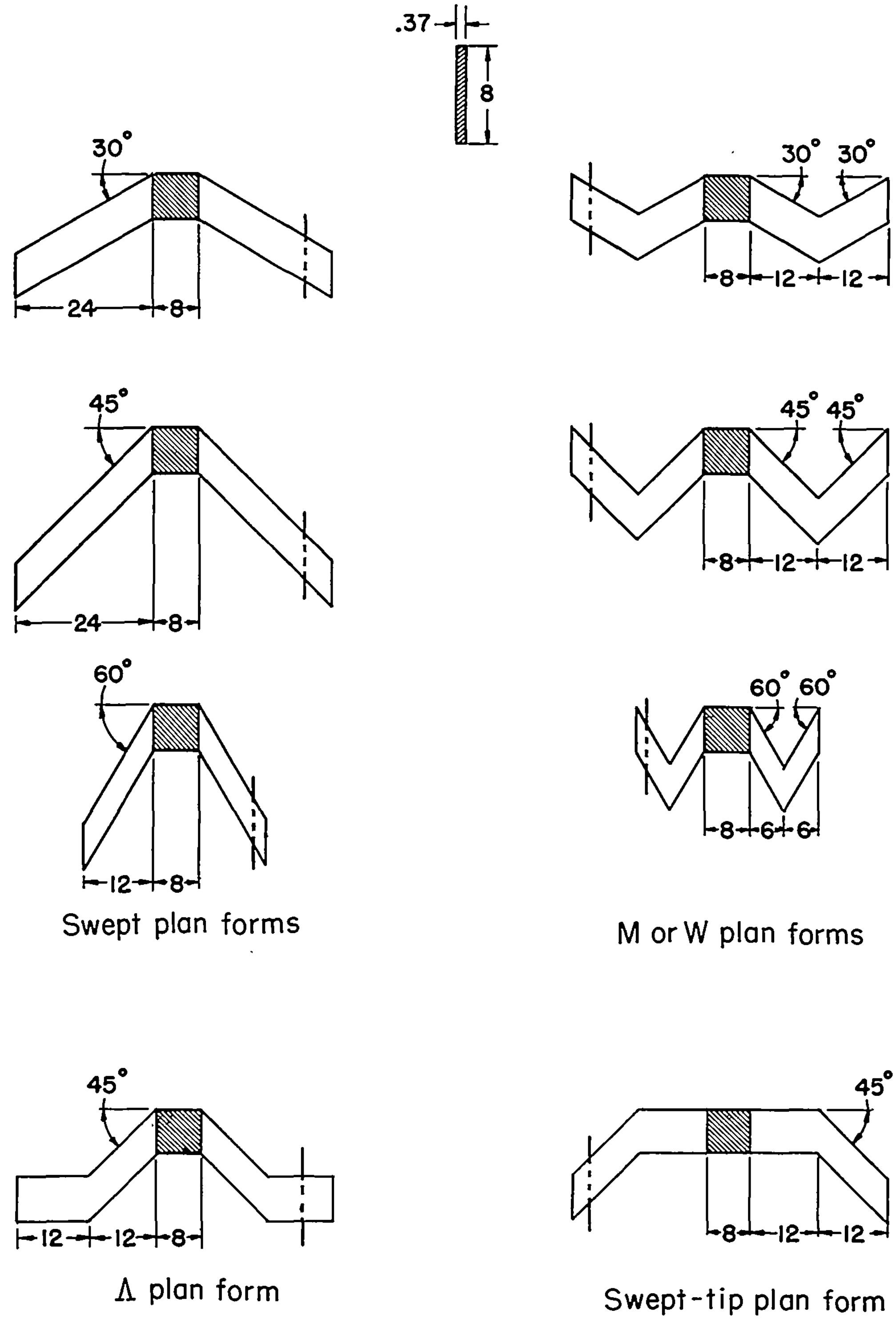


Figure 2.- Details of models.

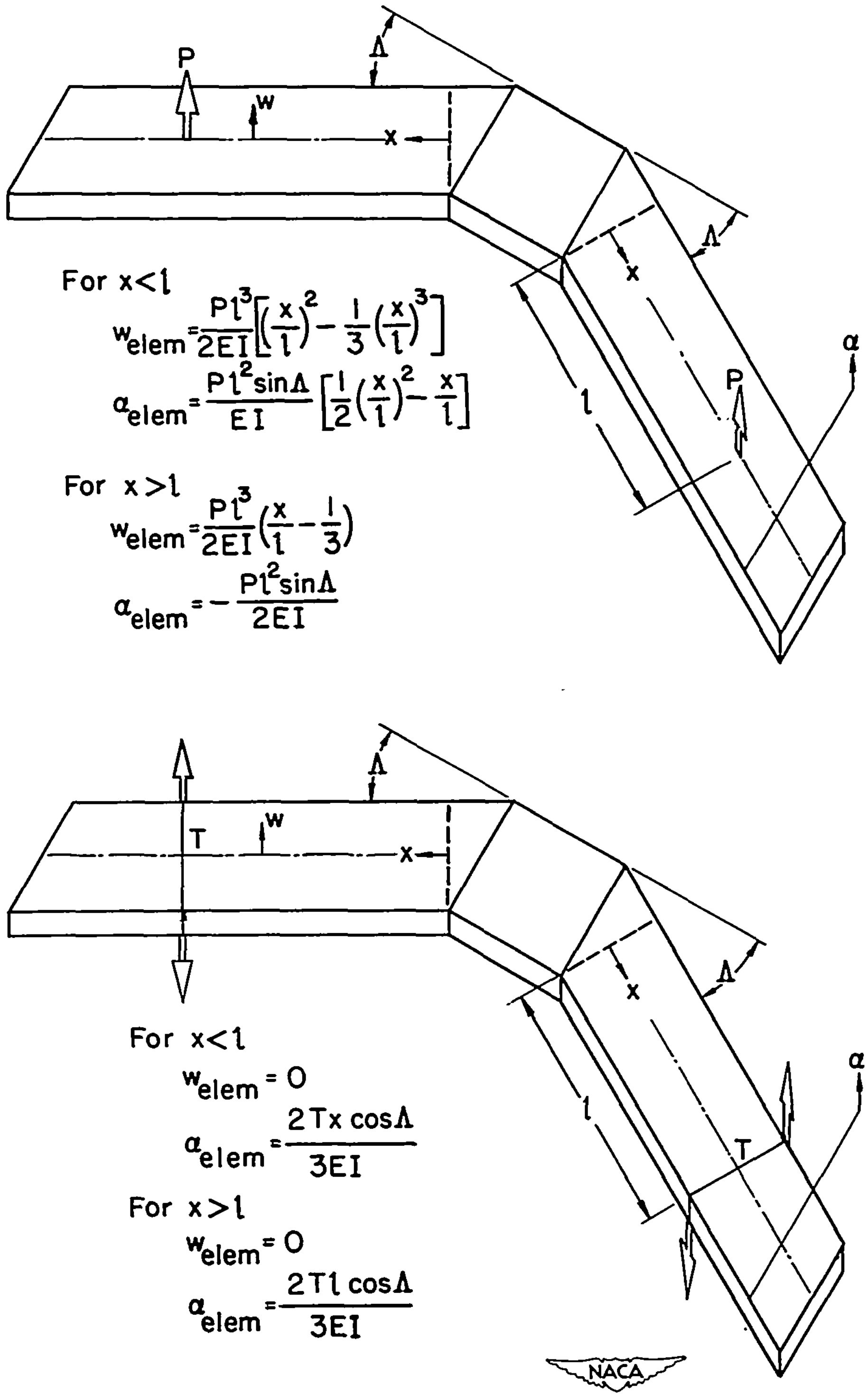
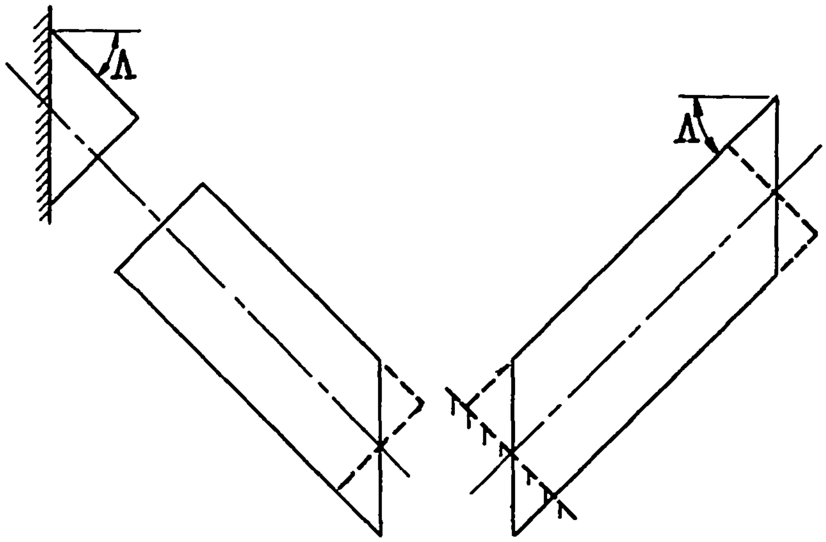
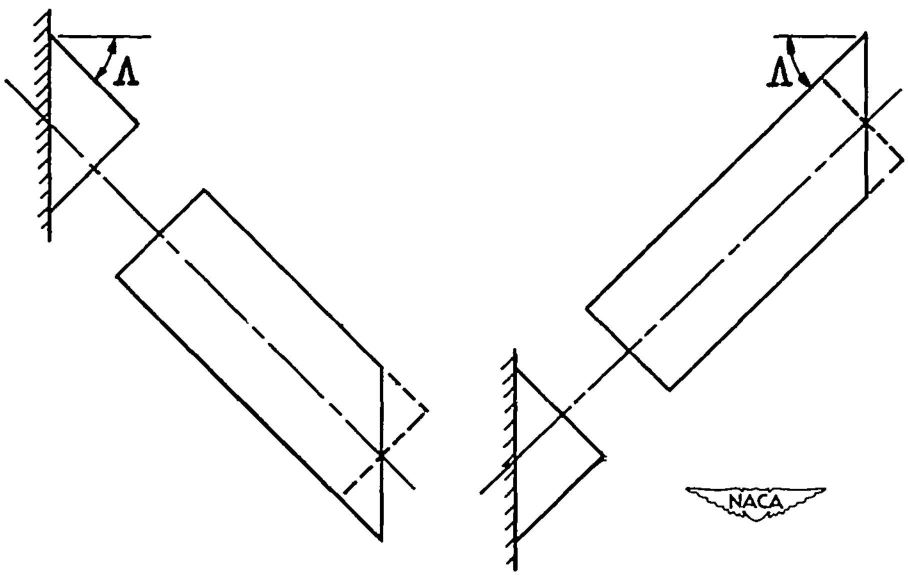


Figure 3.- Application of elementary theory for lift and torque loads.



Structure assumed for theory 1



Structure assumed for theory 2

Figure 4.- Idealization of M or W models.

Figure 5.- Test setup of 60° swept model.

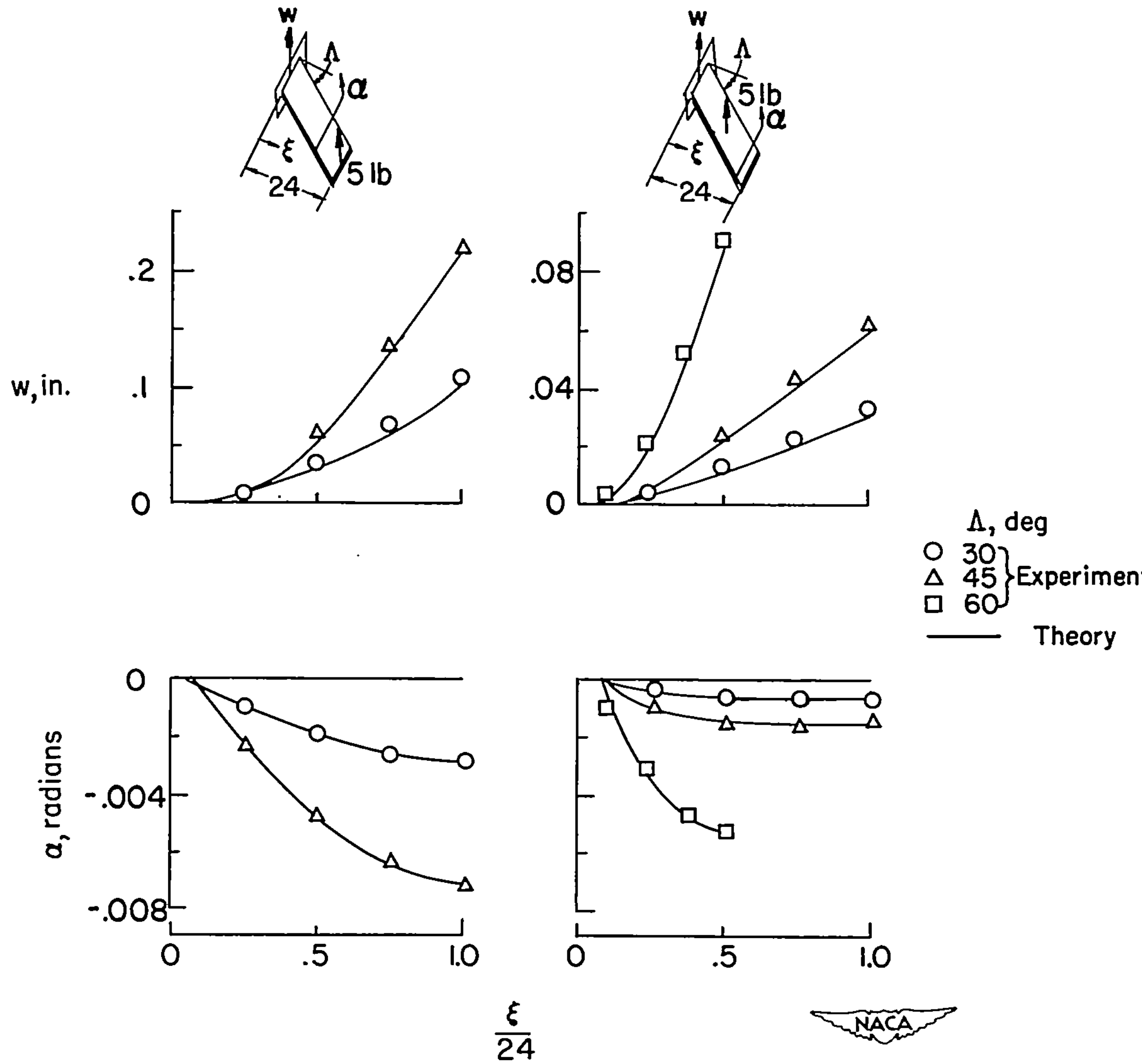


Figure 6.- Deformations of swept models for lift loads.

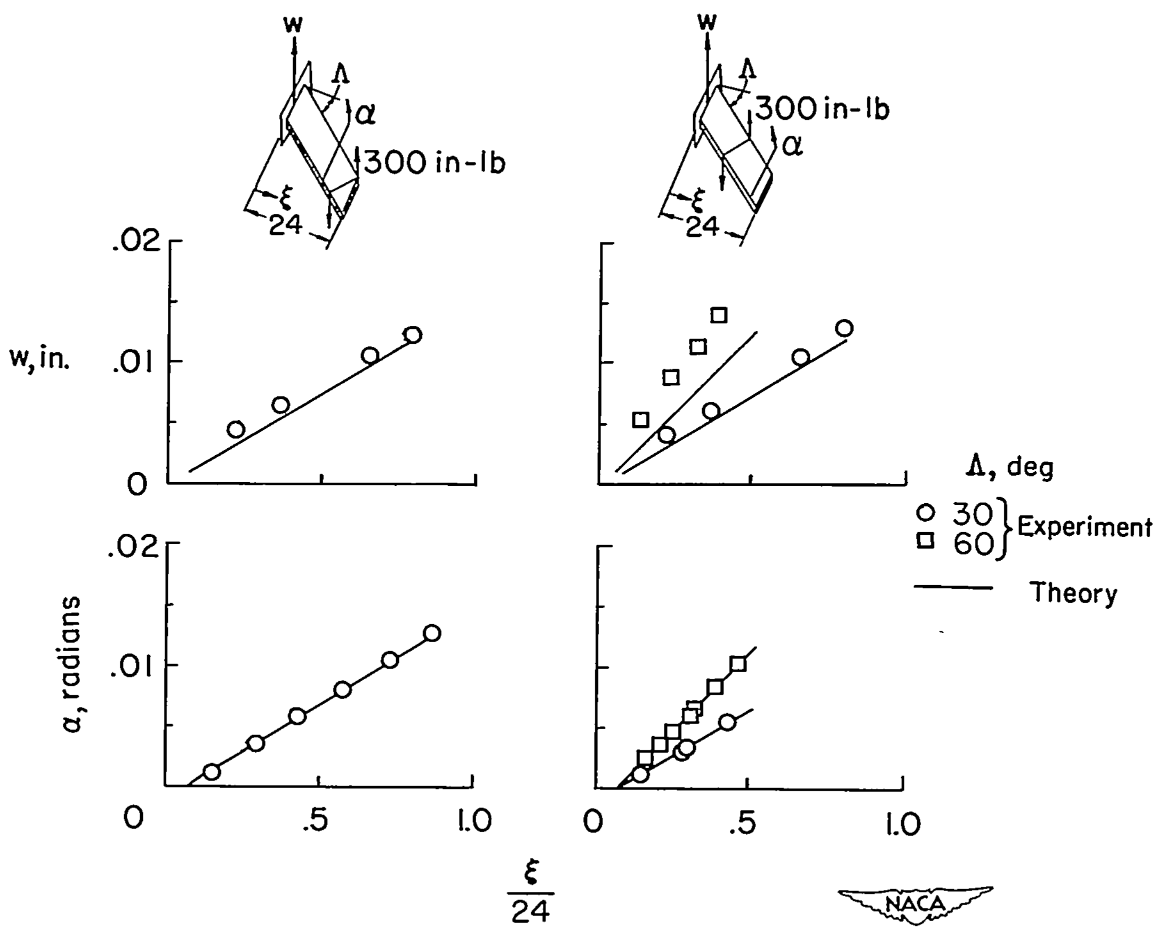


Figure 7.- Deformations of swept models for torque loads.

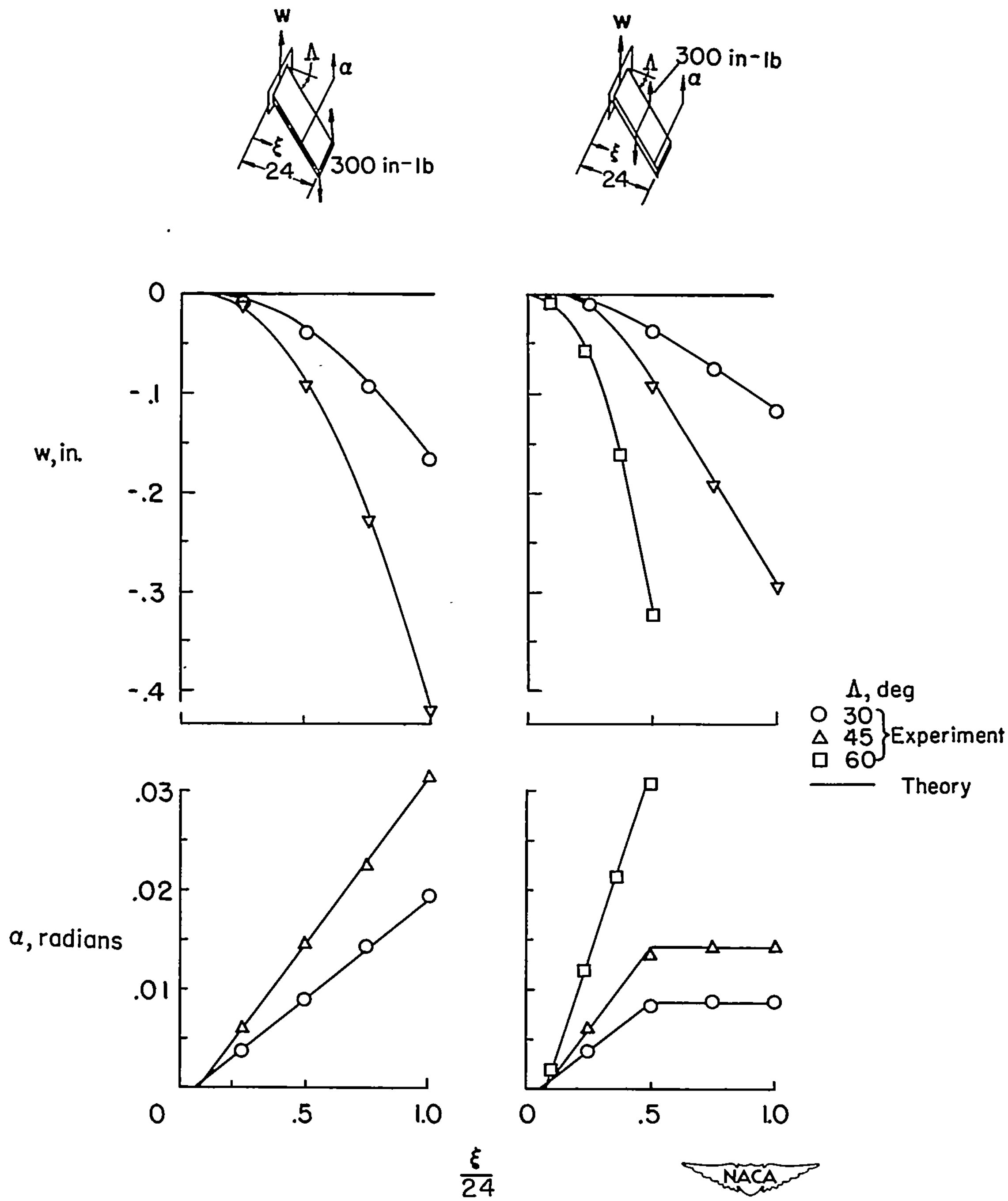


Figure 8.- Deformations of swept models for streamwise torque loads.

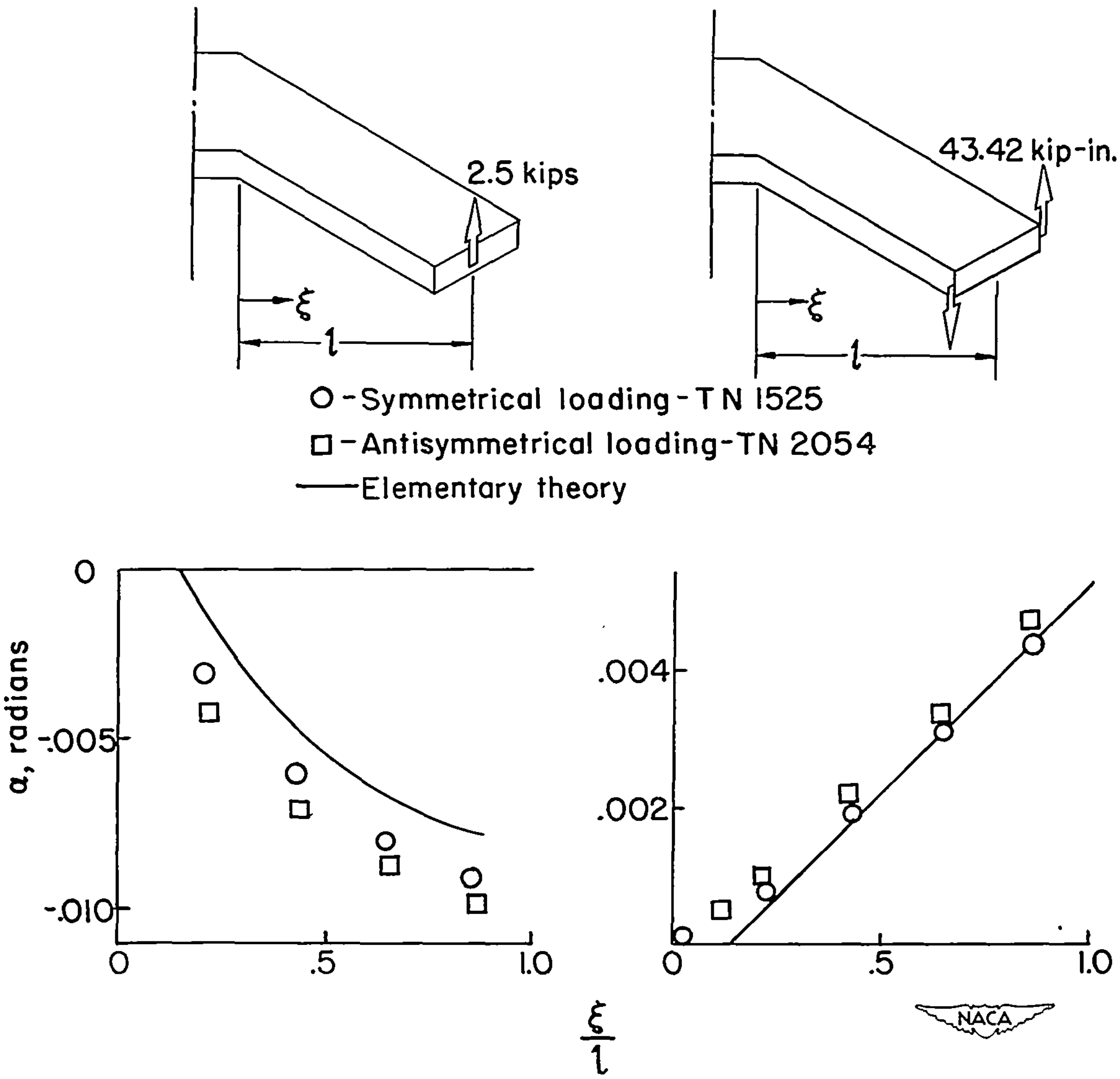


Figure 9 .- Angle of attack of 45° swept box for tip bending and tip twisting loads.

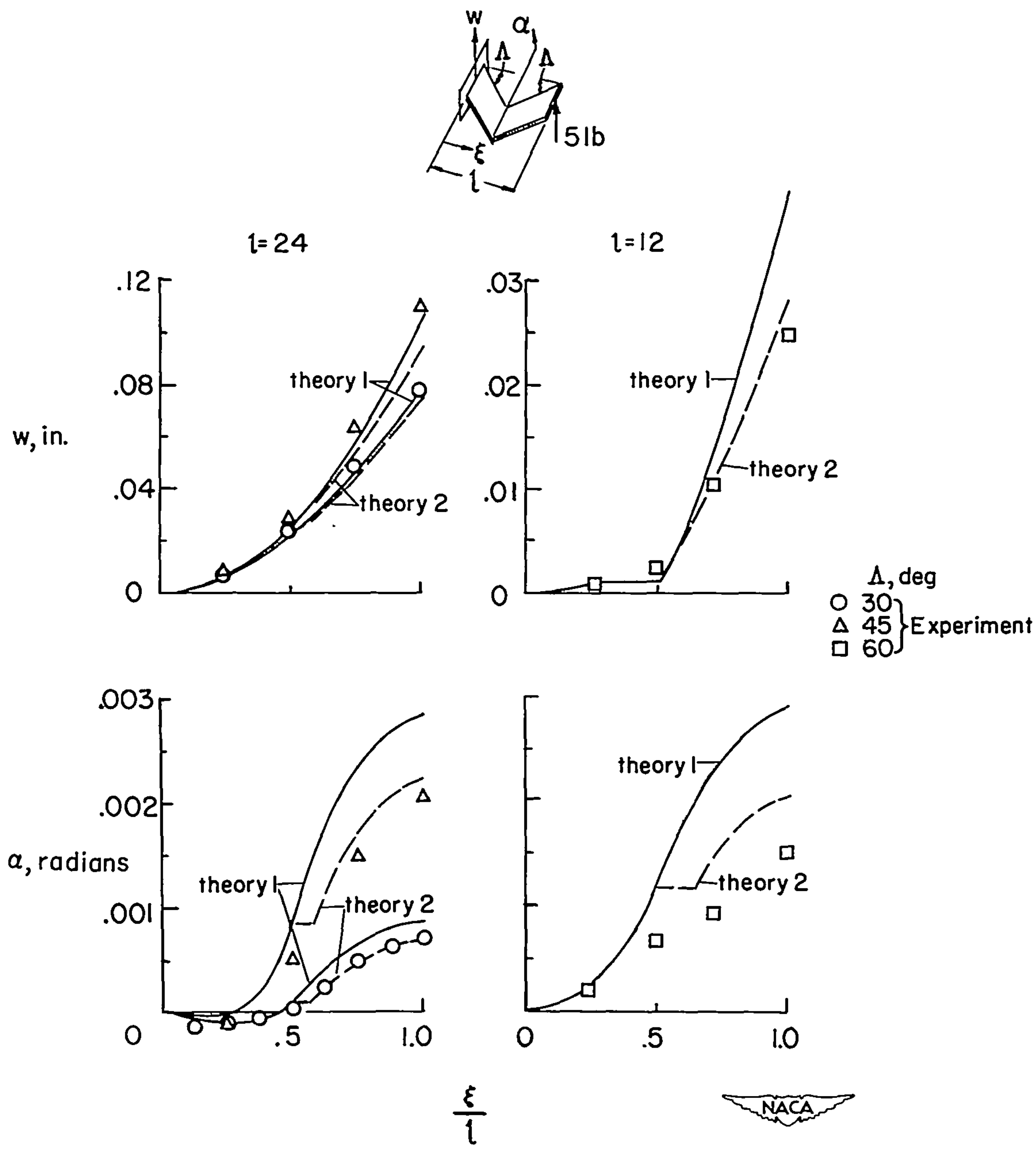


Figure 10.- Deformations of M or W models for lift loads.

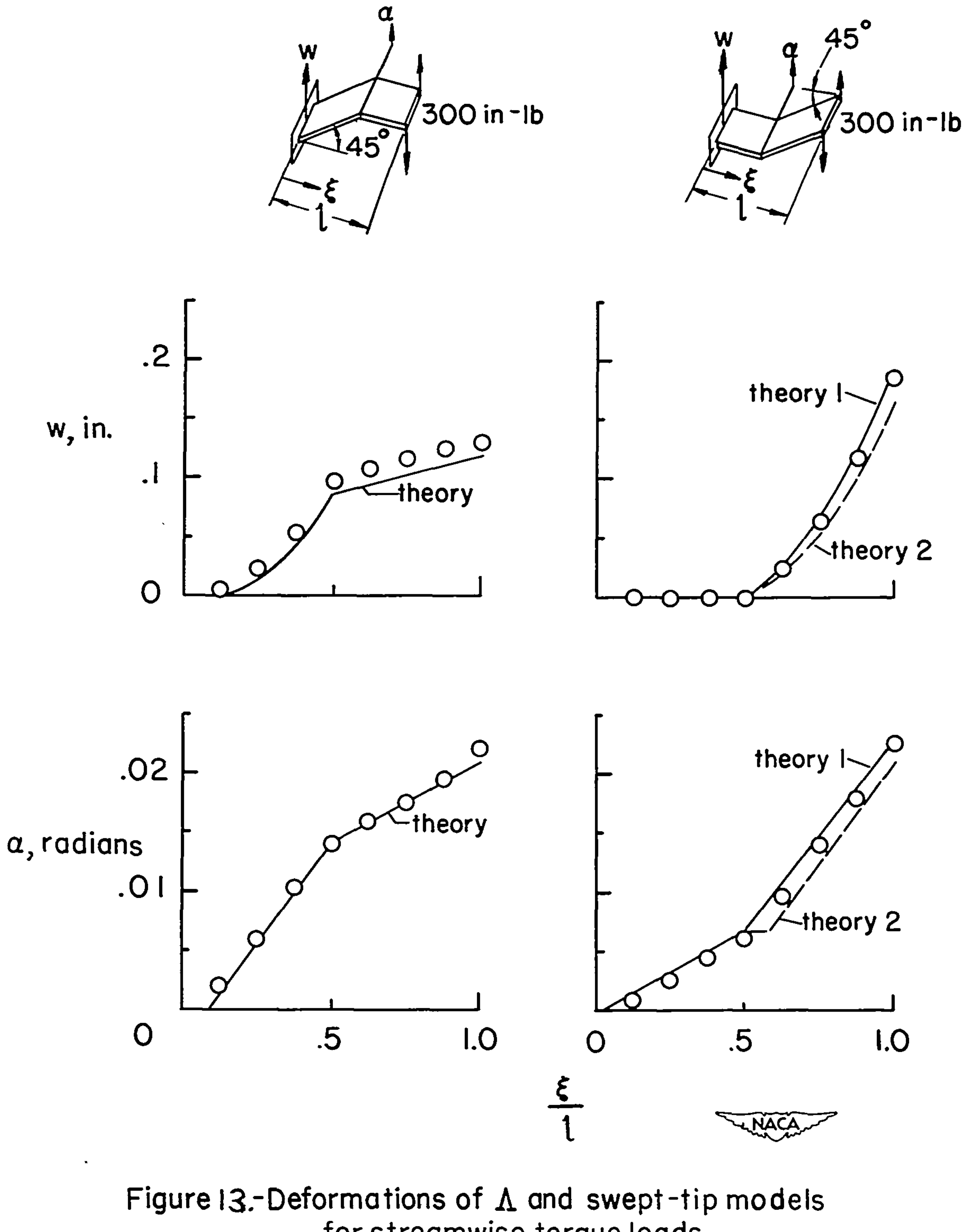


Figure 13.- Deformations of Λ and swept-tip models for streamwise torque loads.

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